Factors affecting spatial resolution

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The theory of spatial resolution has been well-established in various papers dealing with inversion and prestack migration. Nevertheless, there is a continuing flow of papers being published on spatial resolution, in particular in relation to spatial sampling. This poster paper continues the discussion, and deals with various factors affecting spatial resolution.

The theoretically best possible resolution can be predicted using Beylkin's formula. This formula gives answers on the effect of frequency, aperture, offset and acquisition geometry. In this paper these factors are investigated using a single diffractor in a constant-velocity medium. The width of the spatial wavelet resulting from migration of the diffraction event is compared with the predicted resolution. Theoretically, zero-offset data produce the best possible resolution and 3-D shots the worst, with common-offset gathers and cross-spreads scoring intermediate.

The effects of sampling and fold cannot be directly derived from Beylkin's formula, these effects are analyzed by looking at the migration noise rather than at the width of the spatial wavelet. Random coarse sampling may produce somewhat less migration noise than regular coarse sampling, though it cannot be as good as regular dense sampling. The bin-fractionation technique does not achieve higher resolution than conventional sampling.

Generally speaking, increasing fold will not improve the theoretically best possible resolution. However, as noise always has a detrimental effect on the resolvability of events, fold — by reducing noise — will improve resolution in practice. This also applies to migration noise as a product of coarse sampling.

Some theory first. Frame 1 extends an important result from the literature on temporal resolution to spatial resolution: As long as the coverage of the small wavenumbers is complete, it is justified to use the maximum wavenumbers as a measure for spatial resolution.

Maximum wavenumbers can be conveniently determined using Beylkin's formula. This formula was derived as part of a larger objective, i.e., the derivation of migration formulas for single-fold 3-D datasets (also called minimal data sets), which have two varying spatial coordinates ξ_1 and ξ_2, and traveltime t as the third coordinate (for examples see frame 2).

Beylkin's formula (see frame 3) allows the computation of the wavenumber vector $\mathbf{k} = (k_x, k_y, k_z)$ from frequency f and $\nabla_x \phi(\mathbf{x}, \xi)$, the derivative of $\phi(\mathbf{x}, \xi)$ with respect to the output point $\mathbf{x}$.

Lessons from temporal resolution investigations

In a classic paper Kallweit and Wood (Geophysics, 1982) discuss how various criteria (Rayleigh, Ricker, Widess criteria) can be used to describe the width of a wavelet as a measure of temporal resolution. Knapp (Geophysics, 1990) expands on that work and concludes that

temporal resolution is proportional to maximum frequency

(though strictly speaking to bandwidth). Their results can be extended into the realm of spatial resolution, i.e.,

spatial resolution is proportional to maximum wavenumber,

and,

the minimum resolvable distance is inversely proportional to maximum wavenumber.

1

3-D single-fold data sets (minimal data sets)

3-D data set	shot position $\mathbf{x}_s$	receiver position $\mathbf{x}_r$
3-D shot	$(X, Y, 0)$	$(\xi_1, \xi_2, 0)$
3-D receiver	$(\xi_1, \xi_2, 0)$	$(X, Y, 0)$
cross-spread	$(X, \xi_2, 0)$	$(\xi_1, Y, 0)$
common-offset	$(\xi_1 - H_x, \xi_2 - H_y, 0)$	$(\xi_1 + H_x, \xi_2 + H_y, 0)$

X, Y, H_x and H_y are fixed, ξ_1 and ξ_2 vary.

2

Beylkin's formula

Spatial resolution is described by wavenumber spectrum of migration operator

$$\mathbf{k} = f\,\nabla_x \phi(\mathbf{x}, \xi)$$

where

$$\phi(\mathbf{x}, \xi) = \tau(\mathbf{x}, \mathbf{x}_s) + \tau(\mathbf{x}, \mathbf{x}_r)$$

is diffraction traveltime surface and $\xi = (\xi_1, \xi_2)$ describes the two spatial variables of a minimal data set.

Note. If $\mathbf{k}$ is written as

$$\mathbf{k} = \mathbf{k}_s + \mathbf{k}_r,$$

then $\mathbf{k}_s$ and $\mathbf{k}_r$ indicate the direction at $\mathbf{x}$ of the raypath from shot to $\mathbf{x}$, and from $\mathbf{x}$ to receiver, respectively.

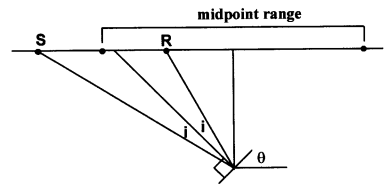
3

The resolution that can be computed from Beylkin's formula is the theoretically best possible resolution. This *potential resolution* depends on geometry, velocity model, source wavelet and output position, and it assumes perfect sampling. In the presence of noise and with less than perfect sampling, the *achievable resolution* will be less good than the potential resolution.

This poster paper is modeled after a poster paper presented at the 1997 SEG Summer Research Workshop in Vail.

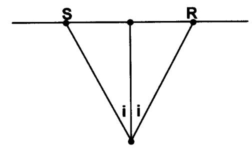
A simple formula for 2-D common-offset gathers. Applying Beylkin's formula to a medium with constant velocity v leads to simple formulas for horizontal and vertical resolution as indicated on the right. Note that the *horizontal* resolution is determined by the farthest shot-receiver pair. The wider the midpoint range (the aperture), the steeper the dips that can be illuminated, and the smaller the minimum resolvable distance in the x-direction R_x. The best *vertical* resolution is determined by the shot-receiver pair which is closest to the output point. For zero-offset, the reflection angle i = 0; then the formulas change into more familiar formulas.

Simple resolution formulas



Horizontal resolution

$$R_x \approx \frac{v}{4 f_{\max} \sin\theta \cos i}$$



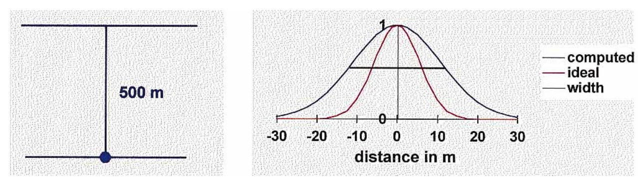
Vertical resolution

$$R_z \approx \frac{v}{4 f_{\max} \cos i}$$

Various factors affecting resolution:
Aperture
Geometry
Fold
Sampling

Procedure to estimate horizontal resolution.

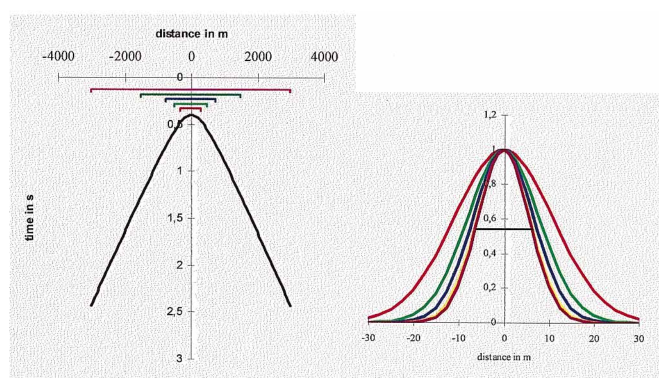
- Model: constant velocity with single diffractor at depth 500 m, source is 50 Hz Ricker wavelet
- For various shot / receiver configurations compute (using true-amplitude migration formula) horizontal spatial wavelet at level of diffractor
  (Ricker wavelet is turned into bell-shaped (Gaussian) wavelet, provided k = 0 is included in measurement configuration)
- Use widths of various spatial wavelets as a measure of spatial resolution
- Compare widths and explain using Beylkin's formula



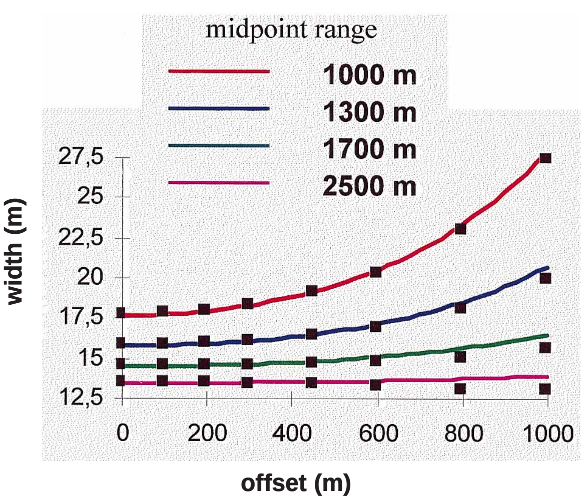
Aperture. Aperture is a major factor determining horizontal resolution. On this page analysis results for a line of sources and receivers are shown (2-D). The diffractor is located below the center of the line. On the right the spatial wavelets are shown for various zero-offset configurations, and in the bottom right corner the measured widths are compared with the prediction from Beylkin's formula. Note the diminishing returns for increasing line lengths: for line lengths larger than 2000 m the widths of the spatial wavelet are virtually the same.

Below, the effect of offset is included. The steepness of the diffraction traveltime curves at the edge of the midpoint range determines the horizontal resolution. The longer the offset the worse the resolution, and the wider the midpoint range the better the resolution.

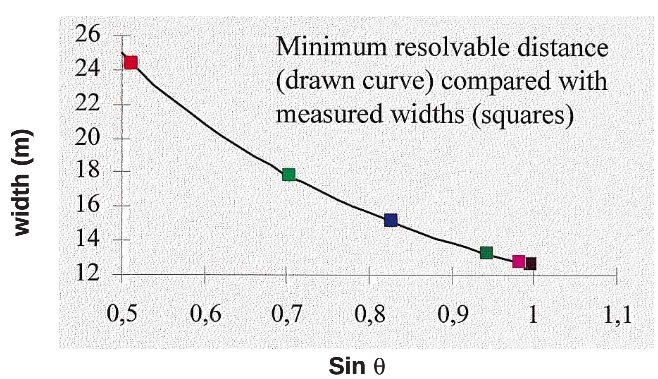
Effect of aperture in zero-offset configuration



Effect of aperture and offset in common-offset configuration

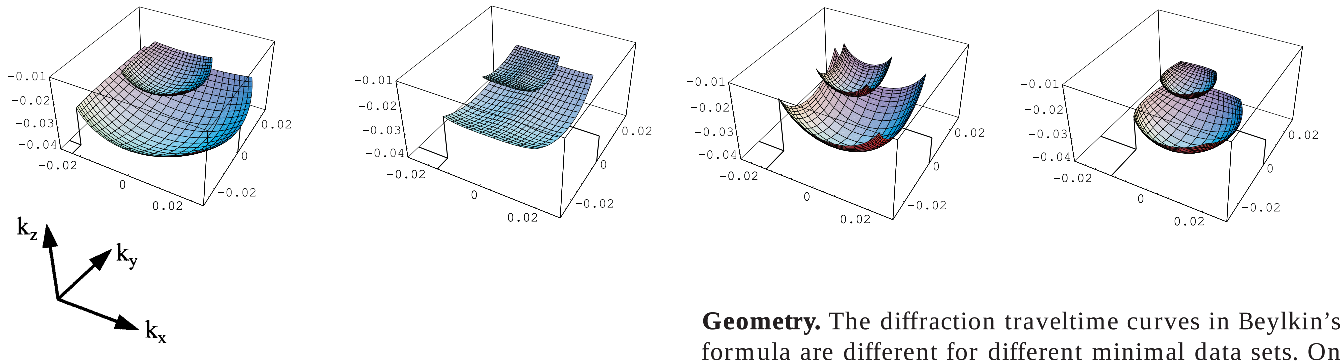


Fit with Beylkin's formula



Wavenumber spectra

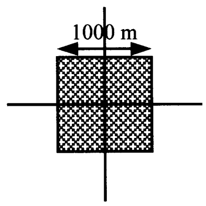
zero offset **offset 1000 m** **cross-spread** **3-D shot**



Configuration for geometry comparisons

Single-fold minimal data sets

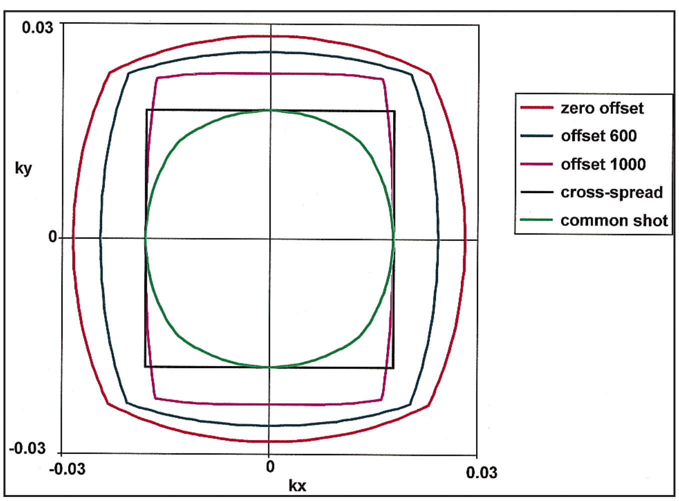
- zero-offset gather
- common-offset gathers (offsets 600, 1000 m)
- cross-spread
- common-shot gather



Same square midpoint area of (-500,500) x (-500,500) m

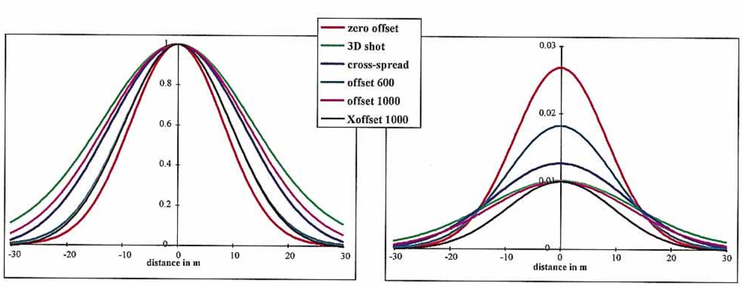
Diffraction in center of midpoint area at depth 500 m

Behavior of k_x and k_y for various minimal data sets



Resolution in x of different minimal data sets

Normalized amplitudes **True amplitudes**



Geometry. The diffraction traveltime curves in Beylkin's formula are different for different minimal data sets. On this page the effect on resolution is investigated for different choices of geometry. The midpoint area of the datasets is the same in all experiments (left).

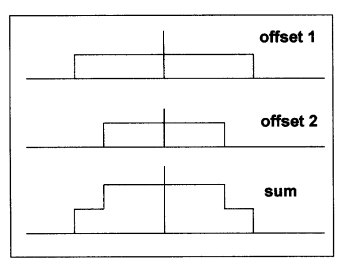
Along the top the wavenumber spectra are shown for four different minimal datasets, for two different frequencies of the input data. The four boxes all have the same size: by comparison, the zero-offset wavenumber spectrum is the widest, and should give the best resolution. Note that a single input frequency covers all horizontal wavenumbers up to some maximum wavenumber: for optimal *horizontal* resolution, a single frequency suffices. On the other hand, a single frequency covers only a limited range in the vertical direction: for optimal *vertical* resolution the broadest bandwidth is required.

On the left the projections of the various wavenumber spectra in (k_x, k_y) are plotted in one figure, allowing a direct comparison. Note the asymmetry in the resolution of the 1000 m offset section: a wider wavenumber range in k_y than in k_x means better resolution in the crossline direction than in the inline direction.

The spatial wavelets shown in the bottom left confirm the predictions that can be made from the computed wavenumber spectra.

Fold. The migration formulas do not cater for overlapping single-fold data sets. As a consequence, the potential resolution of multi-fold data is some average of the potential resolutions of the contributing single-fold data (see below). Yet, fold improves achievable resolution.

Effect of fold



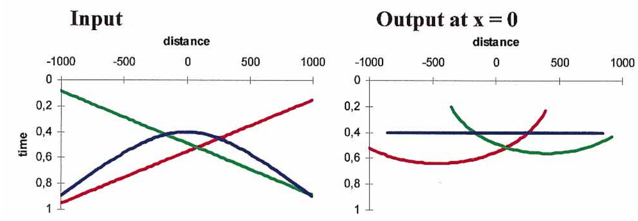
Wavenumber spectrum achievable with common-offset section

Wavenumber spectrum achievable with another comm-offset section (longer offset)

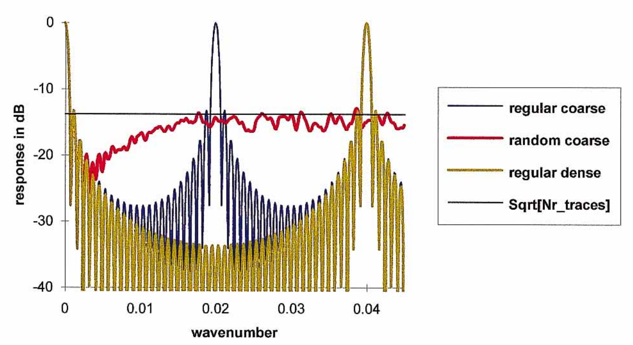
Wavenumber spectrum of two-fold data is some

- Fold tends to skew wavenumber spectrum in output, hence potential resolution is some average of single-fold resolutions (worse than best, better than worst)
- Fold tends to reduce noise, hence tends to

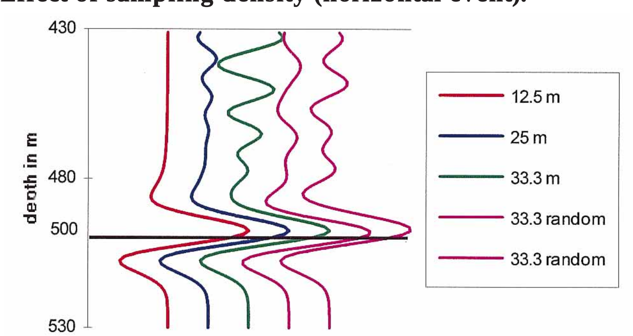
Realignment by migration operator.



Response of migration summation operator.



Effect of sampling density (horizontal event).



Sampling. As pointed out on the first page, potential resolution assumes perfect sampling. It turns out that sampling differences have little influence on the widths of the resulting spatial wavelets. Instead, it is more telling to use the amount of migration noise resulting from inadequate sampling as a yardstick. Along the top part of this page the effect of random and coarse sampling is looked at, along the bottom part the effect of bin fractionation.

Random sampling. To understand the effect of sampling on the migration result (and hence on spatial resolution), it is useful to describe the migration process as a two-step procedure: first, the data are collected along the diffraction traveltime curves corresponding to the output point. This process converts all data contributing to that output point into a new data set, in which the diffraction event from a diffractor in the output point is turned into a horizontal event (see "Output at x = 0" on the left). The second step is to stack all this data into a single trace at the output point.

The response of this second step can be described as a stack operator which depends on sampling (see graphs to the left). Regular sampling leads to an alias (peak) in the operator which does not do any harm for dense regular sampling, but which may pass noise for coarse regular sampling. In that case random sampling can be a better alternative as it avoids the large peak in the response.

The bottom figure on the left confirms that dense sampling gives the best result, and that random coarse sampling may produce less migration noise than regular coarse sampling. These experiments were carried out for 2-D data; for 3-D, the migration of regularly sampled data has already an element of randomness, because the migration process is a function of the distance from input point to output point. These distances are not regularly distributed in 3-D.

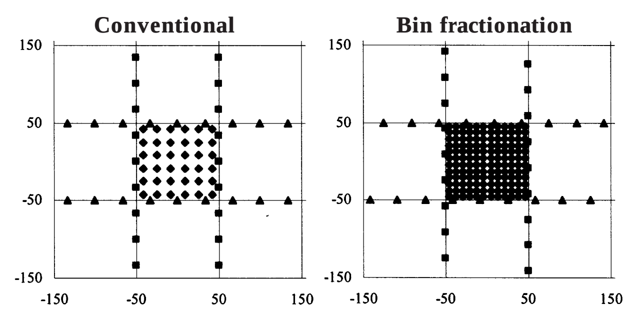
Random sampling (with the right randomness) in the field is difficult to achieve, fortunately, there is no need for it.

Bin fractionation. On the right (top), conventional sampling is compared with sampling using the bin-fractionation technique. The station spacings in the two sampling techniques are the same, but in the bin-fractionation technique the midpoint spacing is smaller. The question is, does this lead to better resolution?

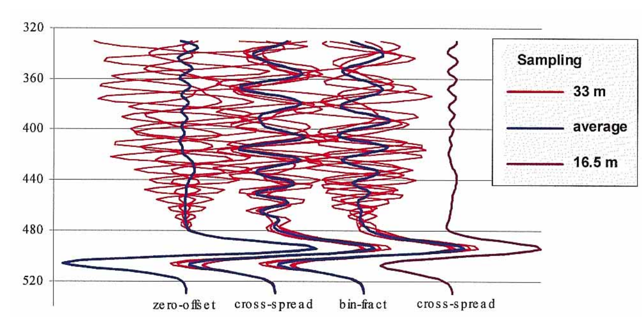
On the right (bottom) I investigate the amount of migration noise using a dipping interface. The orange curves correspond to coarsely sampled data sets, from left to right: four zero-offset data sets, four regularly sampled cross-spreads and four cross-spreads sampled with the bin fractionation technique. When the four zero-offset sections are combined we get a new single-fold zero-offset section with much better sampling, indicated by the blue curve which shows much less migration noise. Combining the other datasets does not lead to better sampled datasets, only to higher fold, and migration noise is now reduced according to rules of fold in both cases. In other words, bin fractionation does not compensate for coarse sampling, even though the midpoint sampling interval has been reduced.

On the far right, the result is shown for a single cross-spread, but now sampled with station spacings that are halved. This curve, and all of the three blue curves represent results for the same number of input traces. Increasing fold only *reduces* migration noise, but finer sampling may *prevent* migration noise.

Cross-spread sampling schemes



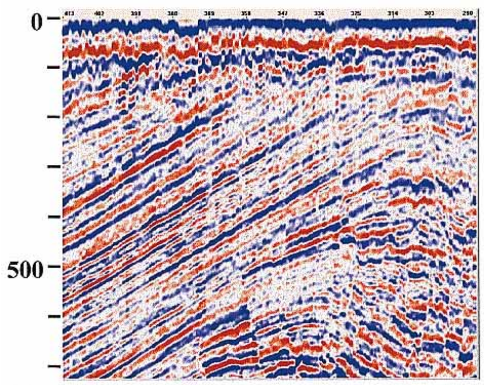
Effect of midpoint grid on migration noise

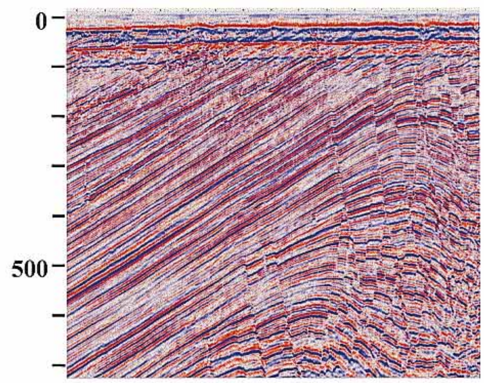


Discussion. Beylkin's formula provides an easy means to compute the theoretically best possible resolution for any single-fold measurement configuration, any velocity model and any source wavelet. In a somewhat disguised form the formula is nothing else than the more familiar formula $k = f / v$, in which v is apparent velocity. The minimum apparent velocity that can be reached is equal to the medium velocity V, and this provides an upper limit to the potential resolution. The maximum frequency f_{max} depends on the source wavelet, hence the *upper limit of spatial resolution* would be reached for $k = f_{max} / V$. Moreover, normally the measurement configuration does not allow acquisition of the smallest apparent velocities, certainly not in all spatial dimensions, and therefore spatial resolution is also limited by the measurement configuration.

Fold is normally needed for noise reduction, fold will also reduce migration noise caused by the effects of coarse sampling, but the best possible resolution can only be reached if the integrands in the migration summation process are properly sampled. Edges in a geometry will produce edge effects which negatively affect the achievable resolution. Therefore, the number of edges in a geometry should be minimized. This means for instance that brick-wall geometries should not be used if maximum resolution is aimed for.

Finally, a "quiz" for the reader: which one of the two seismic sections below had the broader bandwidth and the finer sampling?





Related papers

Beylkin's formula can be found in Beylkin et al. (1985, Spatial resolution of migration algorithms: in Berkhout et al., Eds., *Proceedings of the 14th Internat. Symp. on Acoust. Imag.*). I learnt about Beylkin's formula from the papers by Von Seggern (GEOPHYSICS, 1991 and 1994). Von Seggern (1994) also treated spatial resolution of 3-D data, but his results are often difficult to explain ("counter-intuitive"), because multifold data were used for all computations. Beylkin's formula for spatial resolution formalizes a measurement of resolution which is not unique: Wu and Toksöz (GEOPHYSICS, 1987) and many other authors (e.g., Goulty, *First Break,* October 1997) on crosswell imaging also map spatial information to the wavenumber domain.

Eventually, an expanded version of this paper may be found in GEOPHYSICS. The term "minimal data sets" was coined by Padhi in an internal Shell paper, and recently proposed in the open literature (Padhi and Holley, *TLE*, February 1997).

Random sampling is discussed in Sun et al. (GEOPHYSICS, 1997).

The bin-fractionation technique was first proposed by Cordsen (1993, *Flexi-bin 3-D seismic acquisition method:* CSEG. *Ann. Mtg. Abstracts*), but claims to fame may also be found in a 1994 publication of the US Gas Research Institute, (*Staggered-line 3-D seismic recording*) and in a US Patent (Flentge, 1997, US Patent 5598378.)

A source for much discussion are Neidell's papers on resolution, including the four-part series published in *TLE* (July-October 1997). The conclusions of this paper contradict those of Neidell to some extent.

Conclusions

- Beylkin's formula describes potential resolution, the theoretically best possible resolution
- Potential resolution depends on geometry, velocity model and source wavelet
- The potential resolution can only be achieved with proper sampling and if there is no noise
- The wider the aperture of the surface seismic the better the horizontal resolution
- Vertical resolution is best if the apex of the diffraction traveltime is included in the aperture
- Different single-fold 3-D datasets have different resolutions for the same midpoint area, best for zero offset, worst for 3-D shots
- Fold may reduce potential resolution, but tends to improve achievable resolution by noise reduction
- Coarse sampling reduces achievable resolution
- Random coarse sampling may reduce migration noise
- The bin fractionation technique does not lead to less migration noise than conventional sampling

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From *TLE*, February 1999:

Dear Editor,

In "Factors affecting spatial resolution" (TLE, August 1998) I discuss wavenumber spectra for various minimal data sets (p.1028) and state: "for optimal *horizontal* resolution, a single frequency suffices" and "for optimal *vertical* resolution the broadest bandwidth is required". These statements show that I did not really understand the meaning of the wavenumber spectra. Indeed, Beylkin's formula maps a single frequency into a wide range of wavenumbers. However, this only means that the minimal data set has illuminated a wide range of dip directions (each shot/receiver pair its own direction at the image point). For optimal resolution in all of those directions each source in the minimal data set still needs to be broadband.

Again, a single frequency does generate a wide range of output wavenumbers. Converting back from depth to time these wavenumbers correspond with a wide range of frequencies with the largest frequency equal to the input frequency. For a point scatterer, these output frequencies may produce a rather narrow pulse as shown in Neidell (TLE, September 1997, p.1240, 1242). The width of that pulse decreases with increasing input frequency. However, with a single frequency, the central pulse is surrounded by strong sidelobes, whereas a source wavelet with a smooth frequency spectrum would produce an output pulse without sidelobes.

Gijs J.O. Vermeer