Quasi Monte-Carlo sampling

On: “A quasi-Monte Carlo approach to 3-D migration: Theory”, Y. Sun, G.T. Schuster, and K. Sikorski (May/June 1997, Geophysics, 62, p. 918-928).

Sun et al. discuss the fact that random coarse sampling produces less serious artifacts in migration than regular coarse sampling. This is an interesting phenomenon which also applies to stacking: for high-fold data, regular offset sampling produces the best result, whereas for low-fold data random offset sampling is better. The authors want to use the phenomenon to reduce the amount of computational effort required in prestack migration and to save on acquisition cost of 3D surveys. However, the case they make for these savings carries - in my view - some weaknesses, which I comment upon in the following discussion. At the end of this discussion note I propose an alternative approach to test random sampling versus regular sampling.

As a rationale for their work, the authors state that 4D integrals need to be computed in prestack migration. In actual practice, it is not as bad as this. The fold of our data is in general 30, 60, or 100 perhaps, but not the 10,000 or more which would result from full sampling of all four coordinates. For N-fold data, the actual effort consists of N 2D integrations - one for each time the survey area has been covered -, followed by a simple summation of the single-fold results. Of course, this still involves much computational effort, in particular because all computations have to be repeated for each output point in a 3D volume. Yet, the effort is far less than would be required with 4D integrals.

The authors claim “Our results imply that 3-D prestack migration can be computed more efficiently (by almost an order of magnitude) on Hammersley points … ” and in the Conclusions they state: “Numerical tests demonstrate that … can reduce … by a factor of 4 or more … .” However, for an honest comparison between random coarse sampling and regular coarse sampling, it is also necessary that the regular data set is truly regular. Unfortunately, in the French model example, the final selection of “regular” data is highly irregular, albeit in a systematic way. For these data, the top right graph of Figure 4 does no longer apply. Only a regular equidistant sampling function (a brand-new brush) would produce strong suppression of energy everywhere except in the grating lobes, whereas the sampling function used for Figures 7a and 7b looks more like a worn-out brush, which will have a much wilder response function. This really explains why the images in Figures 7a and 7b look so bad. Although random coarse sampling should produce better results than regular coarse sampling, it has not been demonstrated that as much as a factor of 4 or more can be saved between the two sampling strategies.

The authors also investigate the effect of random coarse sampling as compared to regular coarse sampling on single-fold data with Figures 10 and 11. Unfortunately, the data shown in these two figures are identical, and must have been produced by either random sampling or regular sampling. The authors conclude (based on the actual comparison, I suppose) that “when the dimension of the quadrature problem is as low as $d = 2$, there appears to be very little advantage in using quasi-Monte Carlo migration.” This explanation is not entirely satisfactory, because the test is a genuine test of the effect of Figure 4, which should also apply to single-fold data. This figure suggests a significant difference between the two types of sampling.

In the French model test the authors do not distinguish between reducing fold and reducing sampling density (fold is not even mentioned in the paper). This might be related to their choice of test geometry which hardly bears a relation to geometries used in practice. Yet, an intention of the paper is to demonstrate that the cost of 3D surveys can be lowered in practice. It would have been more convincing, if one of the commonly used geometries, e.g., the orthogonal geometry, would have been chosen as a starting point for the investigation. In the orthogonal geometry sampling density and fold are decoupled: sampling density is determined by the shot and receiver station intervals along the acquisition lines, whereas fold is determined by the distance between the acquisition lines and the shot and receiver spread lengths [inline fold = receiver spread length / (2 * shot-line interval); crossline fold = shot spread length / (2 * receiver line interval); total fold = inline fold * crossline fold].

I would suggest the following approach: first investigate the effect of sampling density on low-fold data (single-fold or 4-fold using very large shot and receiver line intervals), and then increase fold to see whether this would help to reduce migration noise caused by too sparse sampling. For the regular part of the sampling test one could choose shot and receiver station sampling intervals of 33 m to simulate dense sampling, and then select 50 m, 66 m and perhaps even larger intervals for coarse sampling. For the quasi-random part of the sampling test one might select shot and receiver positions in a narrow strip around the same acquisition lines. The point of using very low-fold data for the initial tests is that with noise-free input data, low-fold is sufficient to image the subsurface as long as the sampling density is large enough. These low-fold tests would form a base-line for higher fold tests.

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