Discussion

On: "3-D true-amplitude finite-offset migration," by J. Schleicher, M. Tygel, and P. Hubral (August 1993 GEOPHYSICS, 58, 1112–1126).

In a very interesting paper Schleicher et al. discuss true-amplitude prestack depth migration of single-fold subsets of various 3-D geometries. In the following, I would like to make some remarks, and I would like to present other candidates for inclusion in the list of seismic measurement configurations.

First, it took me a long time before I understood the very special meaning of the term "experiment" in the paper. It always relates to a specific shot/receiver pair and the data collected around it. This means that a single data volume corresponds to a large number of possible experiments, as you can take any shot/receiver pair from that data volume as the basis of an experiment. This concept is of particular interest for the distinction made in the paper between CMPO experiment and ordinary CMP experiment. Given a midpoint, *the two data volumes produced by the experiments are the same*; therefore one might think that they are the result of the same experiment. Yet, in the authors' view an infinite number of different experiments leads to the same data volume.

The data volume of the CMPO experiment has a number of properties which do not make it very suitable for imaging with the stationary-phase method. As a consequence of reciprocity, the data volume is symmetric with respect to the midpoint position, and each illuminated point in the subsurface is illuminated twice (interchanging shot and receiver produces the same trace). Hence, if imaging is to be successful, there should be two zones of stationary phase.

The property of symmetry also applies to any diffraction traveltime surface, so that the difference function τ_F is always symmetric. The symmetry point in τ_F must be an extremum or a saddle point, unless $\tau_F = 0$. The latter situation arises for a horizontal reflector in a constant-velocity medium, then the Huygens surface τ_D coincides with the reflection traveltime surface τ_R. Giving the reflector a small dip, turns τ_F into a surface with a saddle point in the symmetry point (for all "experiments"). This agrees with the statement in the paper that the method of stationary phase is invalid for the ordinary CMP experiment. It may be possible to prove that the symmetry point is always a saddle point (except for the special case $\tau_F = 0$).

For a reflector with constant dip in a constant-velocity medium, τ_F does have closed contours for the CMPO experiment. However, the zone of stationary phase around the image point will in general not be deep enough for wavelets of any length. For the case illustrated in Figure 1 the local extremum of τ_F has a relief of only 2.5 ms, which is not sufficient for application of the method of stationary phase. I expect that the CMPO experiment may lead to reasonable diffraction stack results in very special cases only. (Of course the CMPO experiment is only a theoretical proposition, as it can only be acquired with overlapping areal arrangements of sources and receivers, not quite a common experiment.)

A common seismic experiment, not mentioned in the paper, is the cross-spread experiment (Walton, 1972). It is the basic subset (Vermeer, 1994) of what is probably the most common land geometry, the orthogonal geometry (widely-spaced parallel shotlines orthogonal to widely-spaced parallel receiver lines), and consists of all traces that have a shotline and a receiver line in common. In the terminology used in the paper we have $(S - S_0).(G - G_0) = 0$. For

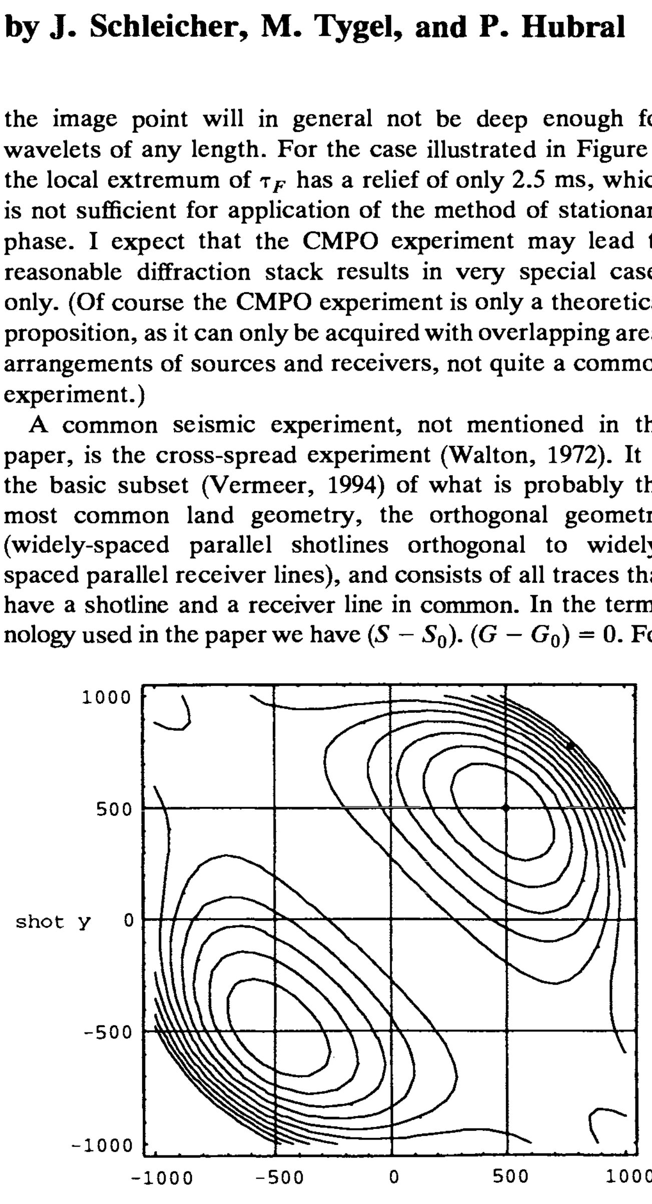

FIG. 1. Contours of difference function τ_F in CMPO experiment for dipping interface in medium with constant velocity 2000 m/s. Dip of reflector is 30°, azimuth 45°. Midpoint located in origin, output point selected at (773, 773). Output point is illuminated by shot at (500, 500). Contour interval 0.5 ms.

shots along lines parallel to the ξ_2-axis and receivers along lines parallel to the ξ_1-axis, this configuration is described by equations (A-1) of the paper, if

$$\underline{\Gamma}_S = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \text{ and } \underline{\Gamma}_G = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}.$$

(Personally, I prefer to think of (ξ_1, ξ_2) as midpoint coordinates of the trace (S, G), then the 1's in the matrices should be 2's, but the above notation is in line with the notation as used in the paper for the CS and the CR experiments.)

Assuming shots and receivers are given by

$$\mathbf{S} = \mathbf{S}_0 + (0, s_2), \text{ and } \mathbf{G} = \mathbf{G}_0 + (g_1, 0),$$

the numerator of the middle term in equation (19) of the paper simplifies to:

$$|\det(\underline{\Gamma}_S^T \underline{\mathbf{N}}_{SM} + \underline{\Gamma}_G^T \underline{\mathbf{N}}_{GM})| = \left| \det \begin{pmatrix} \dfrac{\partial^2 \tau}{\partial g_1 \partial m_1} & \dfrac{\partial^2 \tau}{\partial g_1 \partial m_2} \\ \dfrac{\partial^2 \tau}{\partial s_2 \partial m_1} & \dfrac{\partial^2 \tau}{\partial s_2 \partial m_2} \end{pmatrix} \right|.$$

The reasoning in Schleicher et al. can also be applied easily to the zigzag geometry (Vermeer, 1994). In that case there are two different basic subsets, the zig spreads and the zag spreads for which slightly different formulas apply. It would also be interesting to extend the reasoning to general geometries in which shot and receiver lines are smoothly curved lines rather than straight lines. In case of obstacles, such geometries constitute the best alternative (better than offsetting shots and receivers) to straight line geometries.

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Reply by the authors to the discussion by Gijs J. O. Vermeer

We very much appreciate the competent comments of Mr. Vermeer concerning some topics of our paper. As he points out, the concept of an "experiment," as used by us, is important. We, therefore, take this opportunity to better clarify it. In fact, the term seismic experiment can be given two different meanings: the first one relates, as Mr. Vermeer correctly points out, to one given source-receiver pair, which produces a single seismic trace. The second meaning relates to a certain collection of source-receiver pairs located according to a specific configuration or geometry. In the second case, a seismic experiment produces an organized set of seismic traces, or in other words, a configuration-dependent seismic record section.

The common-mid-point (CMP) and the common-mid-point offset (CMPO) experiments are examples of this second more general meaning of a seismic experiment. The fundamental idea that the source-receiver pairs (**S**, **G**) in a seismic experiment are organized (i.e., located according to a specific configuration geometry) translates mathematically into a *unique parametrization* of their location as a function of a single (vector or scalar) variable or parameter. In simpler words, given a certain value of the parameter, both a source location and a receiver location are uniquely defined. The parameter varies on a set A called the *aperture* of the experiment. For a 3-D seismic experiment, the parameter is a 2-D vector $\boldsymbol{\xi} = (\xi_1, \xi_2)^T$ and the aperture A is a certain bounded area. Also important is the concept of the *data volume* associated with a seismic experiment. In a 3-D seismic experiment this is a 3-D volume of all points (ξ_1, ξ_2, t), in which (ξ_1, ξ_2) belong to A and t is the recording time. The 3-D data volume is a well-defined quantity for each seismic experiment, although, as also correctly pointed out by Mr. Vermeer, several seismic experiments may possess the same or overlapping data volumes.

Mr. Vermeer gives a thorough discussion of the CMPO experiment, acknowledging it as a theoretical proposition, but pointing out its limited practical value. He is very clear and correct in his statements and provides a valuable contribution to the subject.

The remaining comments by Mr. Vermeer refer to the generalization of our work to other useful seismic experiments that may lead to a simplification of our expressions. The formulas presented by him for the cross-spread experiment (Walton, 1972; Vermeer, 1994) are very attractive and absolutely correct. As the surface of a reflector can be covered uniquely with reflection points by a cross-spread experiment (as long as the velocity distribution is not too complex), this experiment indeed defines a complete 3-D seismic data volume. This volume can then be migrated in an amplitude-preserving way with the weight function that includes the configuration matrices and the resulting simplified determinant factor suggested by Mr. Vermeer.

It is interesting to learn that a true-amplitude migration can be applied to the 3-D data volume of a single cross-spread experiment. However, in a conventional 3-D land survey, as Mr. Vermeer also points out, a cross-spread is a subset of the orthogonal geometry (that consists of parallel shot lines orthogonal to parallel receiver lines). Thus, it is surely important to consider whether one can use different subsets of orthogonal-geometry data in a true-amplitude migration.

In principle, the data from many displaced cross-spreads can also be sorted to yield a multitude of common-offset experiments. This sorting has, for instance, the advantage that it automatically provides a larger aperture A for the true-amplitude migration. The migrated image will thus, in turn, suffer from fewer boundary effects.

There is another data subset that takes full advantage of the orthogonal geometry. It pertains to a so-called *3-D cross-profile experiment* as described in Tygel et al. (1992). The configuration matrices for the complete 3-D cross-profile experiment for the case where the parameter vector $\boldsymbol{\xi}$ coincides with the source coordinate vector are specified in Schleicher (1993).

Note, however, that gathering data from widely spaced cross-spreads will not work. In order to guarantee alias-free migrated images (and also true migration amplitudes), the parallel shot and parallel receiver lines must be sufficiently close to each other. However, due to prohibitive costs, this is in general not the case. Therefore, it is not possible in practice to construct (alias-free) three-dimensional subsets of the orthogonal geometry other than cross-spreads.

After a fruitful discussion with Mr. Vermeer via E-mail, we are now also able to present the configuration matrices for the zigzag geometry. If we identify the receiver line with the x-axis, and assume that the sources are displaced along a line that makes an angle of 45° to the receiver line (for an illustration, see Vermeer, 1994), we have

$$\underline{\Gamma}_S = \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}, \quad \underline{\Gamma}_G = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}.$$

These expressions reflect the fact that in a zigzag geometry the source spacing usually is $\sqrt{2}$ times the receiver spacing.

We want to thank Mr. Vermeer for the interest he has shown in our work and for the very constructive remarks he made concerning the application and the generalization of the prestack true-amplitude migration method described by us. What he mentions about the further generalization of our work to curved profiles also looks very promising and surely needs to be investigated.

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Errata

To: "Low- and high-frequency radiation from seismic sources in cased boreholes" (November 1994 GEOPHYSICS, 59, p. 1782)

We regret that an error occurred in printing Equation (7) of this paper. The correct equation is as below.

$$v(r, z, \omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} [-ikA(k)H_0^{(2)}(\sqrt{\omega^2/\alpha^2 - k^2}r) + \sqrt{\omega^2/\beta^2 - k^2}C(k)H_0^{(2)}(\sqrt{\omega^2/\beta^2 - k^2}r)]e^{-ikz}\, dk.$$