NMO stretch for P- and C-waves and its link to resolution, AVO and mute offset

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The effect of NMO stretch on resolution of P-waves

From Vermeer, 2002, p.163, with some modifications:

"Figure 1a illustrates that each shot/receiver pair corresponds to a wavenumber vector **k** = $\mathbf{k}_s$ + $\mathbf{k}_r$, which is normal to the plane illuminated by the shot/receiver pair. For a plane dipping in the *x*-direction with angle θ, **k** = (k_x, k_y, k_z) = $2f$ / v (sin θ cos i, 0, cos θ cos i), where i is the angle of incidence. The factor 1/ (cos θ cos i) is sometimes called the migration stretch factor, or vertical pulse distortion (Tygel et al., 1994). Similarly, the factor 1/ (sin θ cos i) might be called the horizontal pulse distortion. The larger θ the larger k_x, hence the better the horizontal resolution. $\theta_{x,max}$ is determined by the range of input data, or, what is about the same, the migration radius. As argued in Levin (1998), the pulse distortion as a function of θ is only an apparent distortion, because the magnitude of **k** in the θ direction is not affected by it. Only the cos i factor affects all components of **k**, and means a reduction in resolution in all directions. An extensive discussion of these insights is given in Levin (1998)."

The cos i factor represents the ratio between |**k**| for offset data and |**k**| for zero-offset data, i.e., the relative loss in resolution of offset data compared to zero-offset data. In other words, it represents the loss in resolution caused by the NMO stretch effect: f_{NMO} = f cos i. (This can be verified easily for a constant velocity medium and horizontal reflector, but it is plausible that it can be generalized to any medium with continuous velocity; NMO stretch across a velocity discontinuity behaves more complex.) Therefore, the NMO stretch factor S_{fact} is given by

$$S_{fact} = \frac{1}{\cos i} \qquad (1)$$

Regardless whether the data are stacked or migrated, the NMO stretch effect always reduces the resolution of each trace contributing to the final result. Hence the final resolution is determined by an average of the resolution contributions of all traces being stacked or migrated into an output point.

Because it is not simple to find the angle i for all traces contributing to the final result, a pragmatic solution is to describe normal move out with a hyperbola in the CMP domain (like all conventional stacking programs do) and to derive the stretch factor based on that assumption.

Starting from

$$t_0^2 = t^2 - X^2/V^2, \qquad (2)$$

where t_0 is traveltime for zero offset, t is traveltime for offset X, and V is stacking velocity or (what is close) root-mean-square velocity V_{rms}, the stretch factor S_{fact} can be derived as follows (neglecting any dependence of V on t or t_0)

$$S_{fact} = \frac{dt_0}{dt} = \frac{t}{t_0} = \frac{\sqrt{t_0^2 + X^2/V_{rms}^2}}{t_0},$$

or

$$S_{fact} = \sqrt{1 + \frac{X^2}{V_{rms}^2 t_0^2}} \qquad (3)$$

This equation can be used to compute the (mute) offset as a function of t_0 for a given maximum stretch factor. Maximum stretch factors are usually selected in the range from 1.1 till 1.2 (10 to 20% stretch). For S_{fact} = 1.2, $\xi = X/(V_{rms}\ t_0)$ equals 0.66.

Next, the task is to compute the average effect on resolution of a given offset distribution. I will do this for a constant trace density as a function of offset (corresponding to a 2D offset distribution) and for a trace density that increases linearly with offset (as in wide-azimuth 3D offset distributions). Instead of computing the average of cos i to compute the effect, a reasonable approximation is to use the inverse of the stretch factor as given in equation (3). Rather than using X, I will use ξ (offset scaled with $V_{rms}\ t_0$).

The average value of $1/\sqrt{(1+\xi^2)}$ for $0 \le \xi \le \xi_{max}$ is $Arc\sinh(\xi_{max})/\xi_{max}$. Hence, the average stretch factor for 2D data $S2D_{av}$ follows from

$$S2D_{av} = \frac{\xi_{max}}{Arc\sinh(\xi_{max})} \qquad (4)$$

For the 3D case trace density increases linearly with offset, hence the wavenumber contribution of each offset has to be weighted with (scaled) offset ξ, and the average value of $\xi/\sqrt{(1+\xi^2)}$ for $0 \le \xi \le \xi_{max}$ has to be computed in this case. This average value equals $2\ (\sqrt{(1+\xi_{max}^2)} - 1)\ /\ \xi_{max}^2$, so that the average stretch factor for wide-azimuth 3D data $S3D_{av}$ follows from

$$S3D_{av} = \frac{\xi_{max}^2}{2(\sqrt{1+\xi_{max}^2} - 1)} \qquad (5)$$

Taking $\xi = \xi_{max}$ in $S_{fact} = \sqrt{(1 + \xi^2)}$ links the maximum stretch factor via equation (4) to $S2D_{av}$ and via equation (5) to $S3D_{av}$. The average stretch factors for 2D and 3D are plotted as a function of the maximum stretch factor in Figure 2.

Obviously, the average stretch factor for 2D data is much smaller than for the corresponding average stretch factor for 3D data. Interestingly, the two curves are virtually straight lines, which is not immediately obvious from equations (3) – (5). Expansion of the expressions shows that if the maximum stretch factor is written as 1 + x, $S2D_{av} \cong 1 + x/3$ and $S3D_{av} \cong 1 + x/2$. Therefore, if a maximum stretch factor of for instance 1.18 is acceptable for 2D data with an average stretch factor of 1.06, the extra loss in resolution in 3D with average stretch factor of 1.09 is not likely acceptable. For the same loss in resolution due to NMO, the 3D data should have a maximum stretch factor of 1.12 rather than 1.18, i.e., a tighter mute function should be used in processing. In acquisition, it is likely that the same maximum stretch factor in 3D as in 2D should be used to compute the maximum mute offset in order to maintain accuracy of velocity determination and perhaps the ability to carry out AVO analysis as well.

AVO and maximum stretch factor

The relation S_{fact} = 1/cos i provides a direct link between reflection angle and required maximum stretch factor. Figure 3 illustrates the relationship. In the past, simplified amplitude versus angle relationships were used that were valid up till 30°; in that case a maximum stretch factor of 1.16 is sufficient to provide the offset range required for analysis. Nowadays, larger reflection angles are also used providing higher accuracy. For angles up to 40°, a maximum stretch factor of 1.3 has to be accepted. In case AVO analysis is planned for the deepest target, this large required maximum stretch factor leads to extra long offsets. In survey design this may be achieved by acquiring extra long offsets in one direction only (inline or crossline). These long offsets are to be used in AVO analysis and perhaps also in velocity analysis, but should not be used in structural imaging, because they would reduce resolution too much.

Mute offset as a function of depth for a given maximum stretch factor

Equation (3) can be used to compute the mute offset corresponding to a given maximum stretch factor

$$X_{mute} = V_{rms} t_0 \sqrt{S_{fact,\max}^2 - 1} \qquad (6)$$

In a constant velocity medium this would mean

$$X_{mute} = 2z\sqrt{S_{fact,\max}^2 - 1}, \qquad (7)$$

or, mute offset is linearly dependent on depth z. Using Dix' formula to convert V_{rms} and t_0 into interval velocity V_{int} and interval thickness Δz, equation (6) can be written as

$$X_{mute} = 2\sqrt{\sum \frac{\Delta z}{V_{\text{int}}} . \sum V_{\text{int}} \Delta z} \sqrt{S_{fact,\max}^2 - 1} \qquad (8)$$

Depending on the variability of interval velocity as a function of depth, the first square root in equation (8) is somewhat larger than just z, in the order of 0 to 5%. Therefore, mute offset as a function of depth for any fixed maximum stretch factor is always close to linear. An example is shown in Figure 4 for a velocity distribution in some clastic province.

Often, as a rule of thumb, required offset is selected equal to depth. Figure 4 also shows this offset equals depth curve; it corresponds to a maximum stretch factor of about 1.12. However, depending on the area, clean and useful data may also be present for longer offsets. Therefore, the best mute function should be determined from processing tests, bearing also in mind what has been discussed in the first part of this note.

NMO stretch for converted waves

For C-wave data the wavenumber **k** for a shot/receiver pair describing illumination of a reflector at a point **x** in the subsurface can be computed from

$$\mathbf{k} = \mathbf{k}_s + \mathbf{k}_r = f\left(\frac{\mathbf{u}_s}{V_P} + \frac{\mathbf{u}_r}{V_S}\right), \qquad (9)$$

For zero-offset, the unit vectors $\mathbf{u}_s$ and $\mathbf{u}_r$ are both perpendicular to the reflector with $|\mathbf{k}| = f\,(1/V_P + 1/V_S) = f\,(1+\gamma)/V_P$, where $\gamma = V_P / V_S$. For offset data only **k** is perpendicular to the reflector, but now with (see Figure 1b)

$$|\mathbf{k}| = f\left(\frac{\cos\phi}{V_P} + \frac{\cos\sigma}{V_S}\right) = \frac{f}{V_P}\left(\cos\phi + \gamma\cos\sigma\right) \qquad (10)$$

Hence, resolution is reduced for offset data relative to zero-offset data by a factor $(\cos\phi + \gamma\cos\sigma) / (1+\gamma)$, regardless of the dip of the reflector. Again, the loss of resolution is caused by NMO stretch. Therefore, in analogy to the stretch factor for P-waves, the NMO stretch factor for C-waves $S_{fact,C}$ is given by

$$S_{fact,C} = \frac{1+\gamma}{\cos\phi + \gamma\cos\sigma} \qquad (11)$$

Rosales et al. (2008) show that the angles ϕ and σ can be expressed in the half-aperture angle θ as follows

$$\tan\phi = \frac{\gamma\sin 2\theta}{1+\gamma\cos 2\theta} \qquad (12a)$$

and

$$\tan\sigma = \frac{\sin 2\theta}{\gamma + \cos 2\theta} \qquad (12b)$$

Substituting equations (12a) and (12b) into (11) gives the C-wave stretch factor expressed in half-aperture angle θ and γ

$$S_{fact,C} = \frac{1+\gamma}{\sqrt{1+\gamma^2 + 2\gamma\cos 2\theta}} \qquad (13)$$

(The relationship $(\cos\phi + \gamma\cos\sigma) = \sqrt{(1+\gamma^2 + 2\gamma\cos 2\theta)}$ was already used in Miao et al. (2005), but without giving a derivation.) Note that equation (13) is a generalization of equation (1), because it is also valid for $\gamma = 1$. Figure 5 illustrates the stretch factor as a function of half-aperture angle for a number of different choices of γ. Note that for the same aperture angle the C-wave stretch factor is always smaller than the P-wave stretch factor.

References

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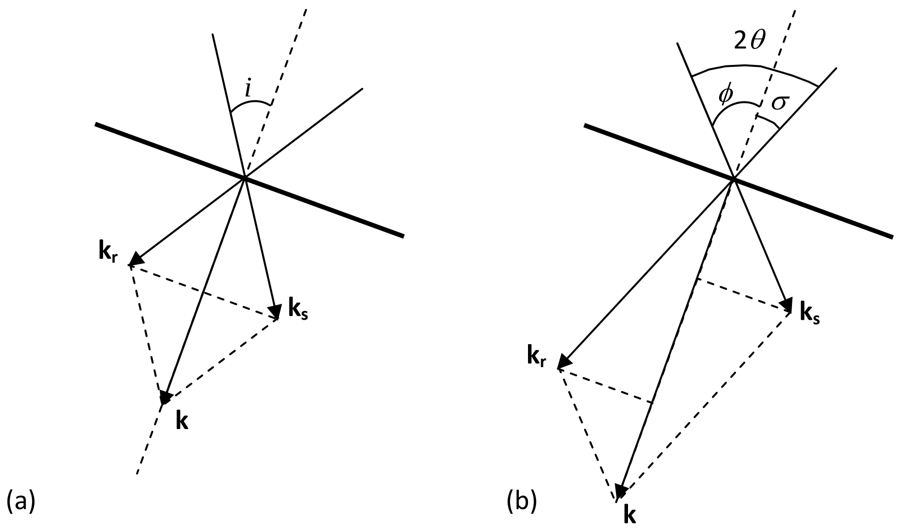

Fig. 1. Illustration of derivation of NMO stretch factor. (a) for P-waves, (b) for converted waves.

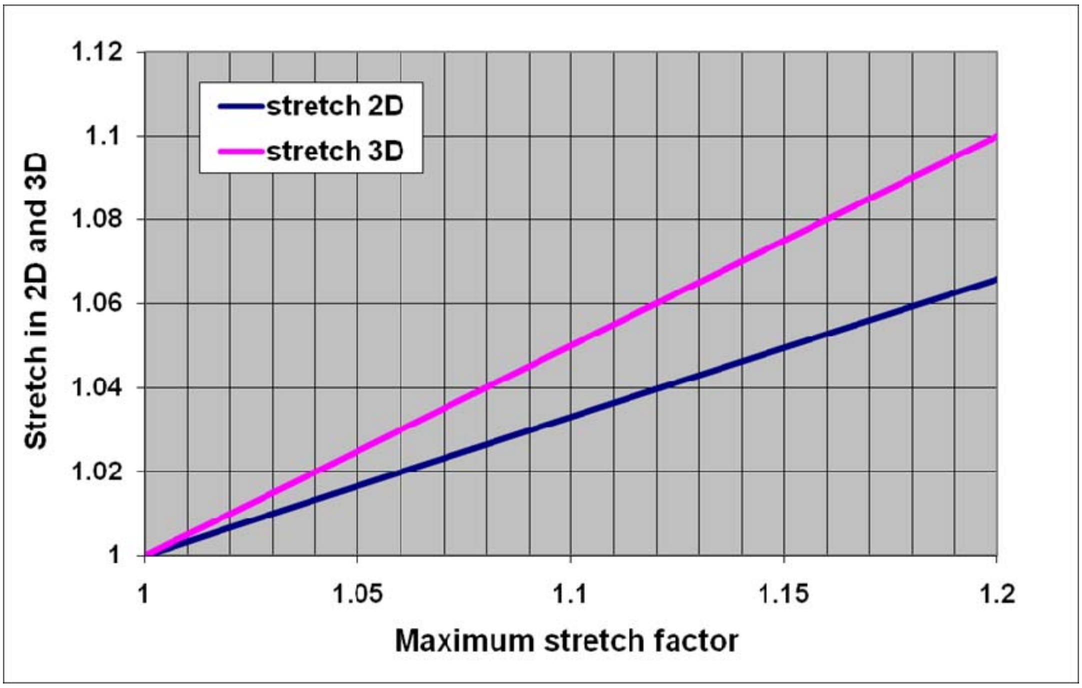

Fig. 2. Average stretch factors for 2D data and wide-azimuth 3D data as a function of the maximum stretch factor.

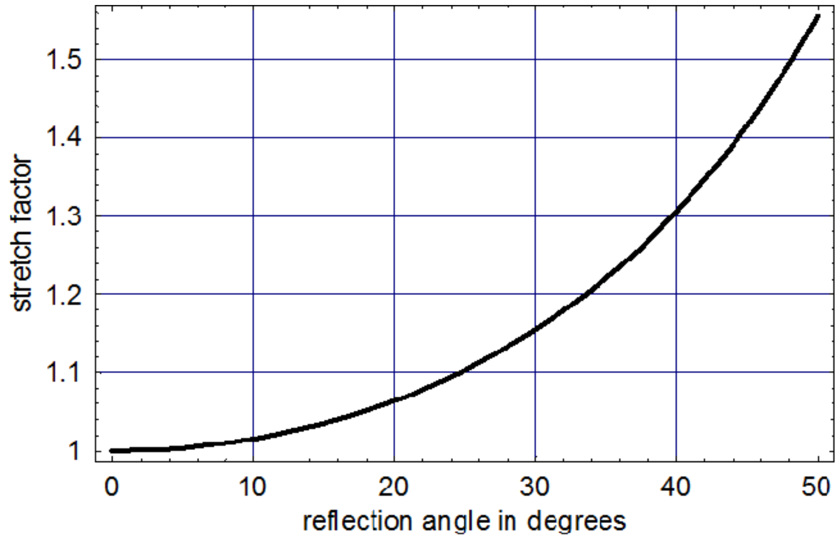

Fig. 3. Stretch factor as function of reflection angle.

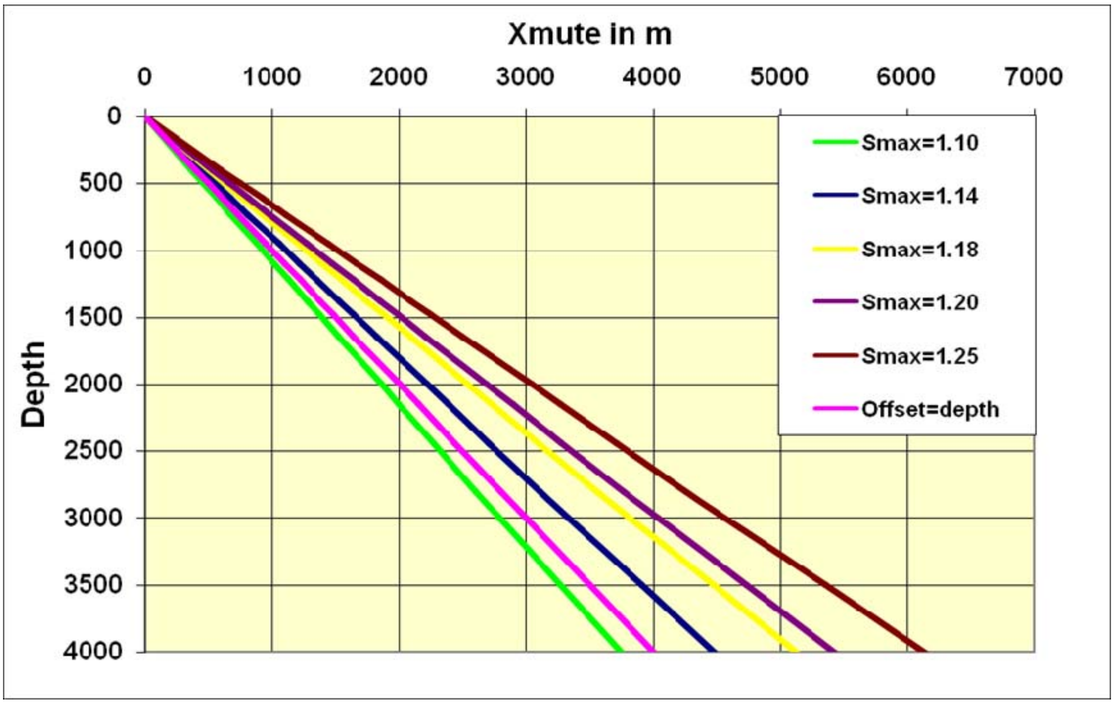

Fig. 4. Mute offset as function of depth for various maximum stretch factors.

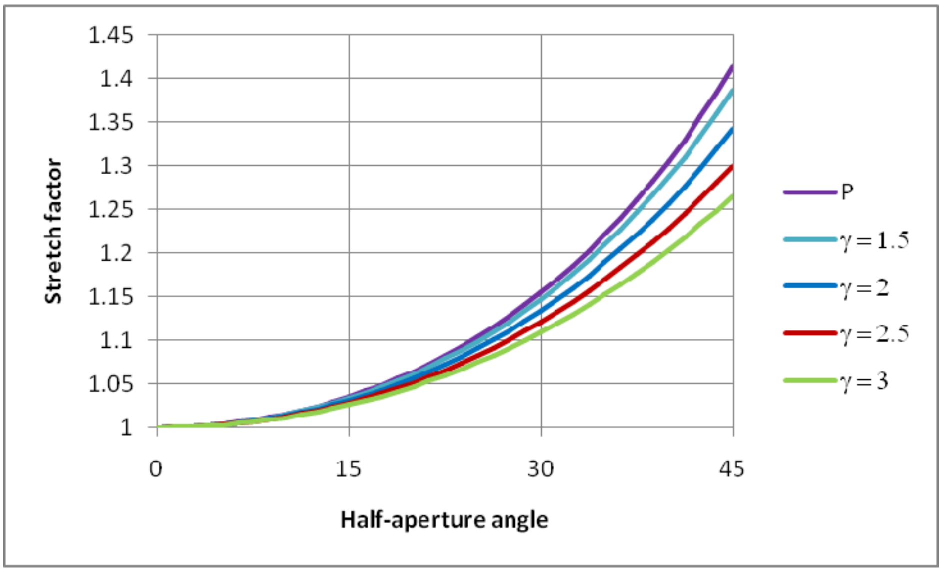

Fig. 5. NMO stretch factor as a function of half-aperture angle for various choices of γ. The curve labeled "P" is the same as in Figure 3.