On: "Unambiguous signal recovery above the Nyquist using random-sample-interval imaging" (R.D. Wisecup, Geophysics, 63, 1997, 763-771)

Wisecup (1997) claims that “a simple, exact sample-mapping methodology, random-sample-interval imaging, can be used to overcome aliasing in many of the processes currently used for the imaging of seismic data.” Other statements are that “increased equipment costs are incurred due to the presumed requirement for antialias filter circuitry in the recording instruments”, and “many antialias strategies currently in use may be inappropriate”, and many more such remarks. The question comes up whether this random-sample-interval imaging (RSI^2) is really a promising method for signal recovery beyond Nyquist.

To address this question I'll start with some remarks about sampling and aliasing, followed by a discussion of the theory of RSI^2 and its implementation.

Numerous papers have been written about the theory of sampling and aliasing, both for one-dimensional sampling (e.g., Nyquist, 1928, Shannon, 1949) and for N-dimensional sampling (Petersen and Middleton, 1962). Jerri (1977, also referenced in Wisecup, 1997) gives a comprehensive overview of all those theories, including sampling theories for stochastic fields, and mentions as many as 248 other papers. The theory of most interest to the seismic world with its deterministic functions is well covered by the N-dimensional sampling theorem by Petersen and Middleton. This theorem is used in Bardan (1987) to overcome aliasing in 3-dimensional signals, in Vermeer (1990) to explain the sampling paradox (aliasing in midpoint and offset domains even if there is no aliasing in shot and receiver domains), and in Bardan (1997) to propose alternative sampling strategies. A recent generalization of the N-dimensional sampling theorem of Petersen and Middleton was given by Soubaras (1997). The Soubaras theorem allows the reconstruction of energy above Nyquist as long as the sampling of the original (continuous) frequency spectrum does not lead to overlapping energy in the sampled frequency spectrum.

Often, our seismic data is spatially aliased, and therefore there is great interest in finding ways of minimizing damage due to aliasing. Sometimes data is perceived to be aliased, while it is not, as in the case of 2D seismic data which may be well-sampled in shot- and receiver domains, but seems to be aliased in midpoint and offset domains. Another such example may occur for seismic data in the CMP: if dips are all increasing with increasing offset, then empty space in the f-k domain may be used to overcome the perceived aliasing (as for instance in Fig. 2 in Wisecup). For such examples it is easy to come up with de-aliasing techniques. In more difficult situations, with actual aliasing of the signal, de-aliasing techniques are often based on an assumption of a limited number of events with constant or smoothly varying signal shape. On synthetic data with a limited number of events such techniques may work perfectly, and on real data aliasing damage may be reduced in very impressive ways, because these techniques are very effective in reducing the damage on the strongest and most conspicuous events. Another reason for successful interpolation can be that the input signal satisfies the Soubaras theorem. However, none of the techniques invalidates N-dimensional sampling theory.

The interesting feature about the RSI^2 technique is that it does not assume a limited number of events. Any number of events is acceptable. However, it does assume that the signal is sampled at (more or less) random positions in different sampling realizations. Implicitly, this means that the same signal should occur on all traces. The synthetic data examples in Wisecup (1997) show exactly this important characteristic: the signal is the same, but time-shifted on all traces. It is not at all surprising, nor a new invention that treating such data with the RSI² method is able to reconstruct the original signal. A very clear explanation of what happens is for instance given in Figure 1 reproduced from Ronen (1987).

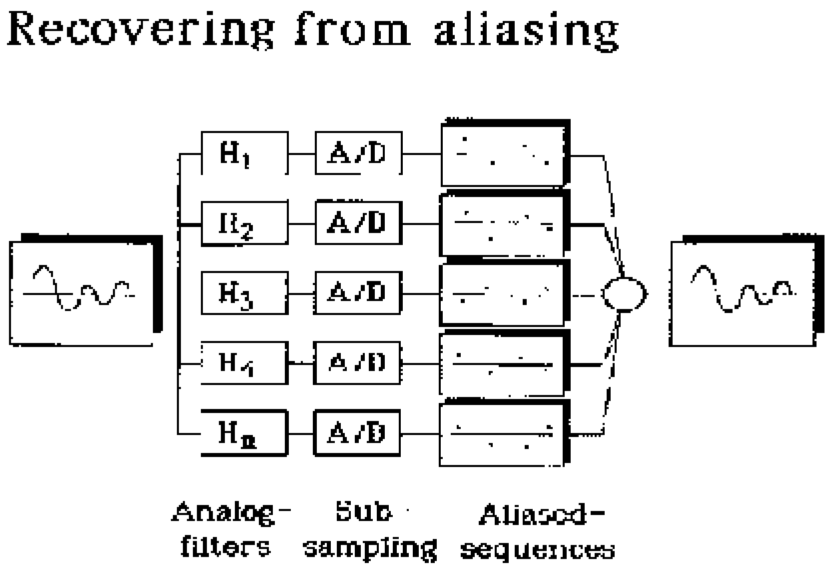

Fig. 1 Overcoming aliasing of a one-dimensional signal. The signal at the left is filtered by five filters H1, ..., H5, before sampling by the analog-to-digital (A/D) converters. The data are five different aliased sequences. The original signal cannot be recovered from anyone sequence alone, but the combination of them may be sufficient.

In conventional (well-seasoned) signal processing, consecutive samples of sampled band-limited data are treated as entities that belong together. For instance, they can be used to make estimates of the amplitude of the signal at any point between the sample positions by some kind of interpolation. However, in the RSI² technique this is not possible, the data may be heavily aliased, and then simple interpolation techniques will lead to wrong answers. Instead, the RSI² technique treats every sample as a separate entity. This does not only allow the recuperation of lost frequencies if the signal is the same across the input traces, but it also ensures the creation of high frequencies if the signal is not the same across the input traces.

The validity of the latter statement is illustrated in the paper: Trace 2 in Figure 8a is the RSI² result for perfectly identical signals, and the result is nearly perfect. Trace 2 in Figure 7a is the RSI² result for signals suffering from stretch. Now the samples of the original signal are placed out of sequence along the new time axis, and jitter is the result. This jitter represents generated high frequencies, which have little energy in this case, but may become much stronger if we not only deal with stretch, but also with noise. In the case of noise, - e.g., multiples running obliquely through the primary signal - each sample has a signal and a noise part. The signal part may behave smoothly, but then the noise part will cause severe jitter, i.e., spurious high frequencies, because consecutive samples on output stem from different input traces. The author states that the results of the real data experiments demonstrate robust performance in the presence of noise, but it is an omission that the effect of noise is not analyzed more thoroughly with synthetic data and with displays as Figures 7 and 8.

I did not yet discuss the actual implementation of the RSI² technique. When the NMO correction is applied to a sample, it will not fall exactly on the location of the finer output sampling grid. Eventually, a number of samples will be spread out around each output sample position, and then they will have to be collected and turned into a single output sample. This is achieved in the paper by the application of a short running mix of the output samples. Essentially, this technique is very much akin to what is called binning in spatial sampling. In a recent paper, Beasley and Mobley (1997) discuss the aliasing caused by binning the DMO operator, and they argue that according to sampling theory, the individual samples have to be spread out over an areal grid of sample positions using a two-dimensional sinc function. And indeed, this approach leads to considerably improved results. Similarly, a better resampling procedure for each output trace of the RSI² technique would be to apply a one-dimensional sinc to each individualized sample. But even this technique would lead to spurious high frequencies if the output sampling interval would be smaller than the input sampling interval and if the signal would not be the same across the traces.

It may be clear from the foregoing that I do not think that the RSI² technique - albeit interesting - is an attractive de-aliasing procedure. My point is further illustrated by the real data examples in the paper. The RSI² result in Figure 10d shows indeed similar frequency content as Figure 10a. However, much of the high frequencies are spurious as is clear from a closer inspection of the figures. With foreshortening, Figure 10a shows a very nice and sharp, nearly horizontal event (the first black loop) running all across the display, whereas in Figure 10d this event is not nearly as sharp. A detailed trace-to-trace comparison shows that many traces in Figure 10d are quite different from Figure 10a. In other words: the high frequencies are there, but they are spurious, at least partially.

In conclusion: the RSI² technique works fine if the signal is constant, and if there is no noise; if there is noise, smoothing in one form or another is necessary to get rid of the spurious high frequencies, and with this technique it is not possible to "unambiguously" recover the signal above Nyquist lost by coarse sampling. For some, the old building of N-dimensional sampling theory might appear to show some cracks, but closer inspection should reveal a reliable construction which will outlast all of us. I do not recommend to do away with anti-alias filters for situations where there is significant energy above the Nyquist frequency.

Note: Implicitly, this discussion note also addresses "Huygens' imaging" promoted in Neidell (1997). Neidell's technique is based on the same idea of individualized samples gathered in tiny bins. In the RSI² technique the samples undergo NMO correction, whereas in Huygens' imaging the samples are corrected according to the diffraction traveltime function.

Gijs J.O. Vermeer

REFERENCES

Bardan, V., 1987, Trace interpolation in seismic data processing: Geophys. Prosp., **35**, 343 - 358.

Bardan, V., 1997, A hexagonal sampling grid for 3D recording and processing of 3D seismic data: Geophys. Prosp., **45**, 819 - 830.

Beasley, C.J., and Mobley, E., 1997, Spatial dealiasing of 3-D DMO: 67th Ann. Internat. Mtg. Soc. Expl. Geophys., Expanded Abstracts, paper # SP3.7.

Jerri, A.J., 1977, The Shannon sampling theorem - Its various extensions and applications: A tutorial review: Proc., Inst. Electr. Electron. Eng., **65**, 1565 - 1596.

Neidell, N.S., 1997, Perceptions in seismic imaging - Part 4: Resolution considerations in imaging propagation media as distinct from wavefields: The Leading Edge, **16**, 1412 - 1415.

Nyquist, H., 1928, Certain topics in telegraph transmission theory: Trans., Am. Inst. Elec. Eng., **47**, 617 - 644.

Petersen, D.P., and Middleton, D., 1962, Sampling and reconstruction of wave-number limited functions in N-dimensional Euclidean spaces: Information and Control, **5**, 279 - 323.

Ronen, J., 1987, Wave-equation trace interpolation: Geophysics, **52**, 973 - 984.

Shannon, C.E., 1949, Communication in the presence of noise: Proc., Inst. Radio Eng., **37**, 10 - 21.

Soubaras, R., 1997, Spatial interpolation of aliased seismic data: 67th Ann. Internat. Mtg. Soc. Expl. Geophys., Expanded Abstracts, paper # SP4.11.

Vermeer, G.J.O., 1990, Seismic wavefield sampling: Soc. Expl. Geophys.

Wisecup, R.D., 1998, Unambiguous signal recovery above the Nyquist using random-sample-interval imaging: Geophysics, **63**, 763-771.

Reply to reply: RSI^2 still interesting, but not attractive

The above note discussing Wisecup's paper was printed in the March/April 1999 issue of Geophysics, p. 632-636, together with the reply by the author. To my taste this reply is a nice step in a scientific discussion which should eventually lead to agreement on at least some points, but unfortunately, Geophysics has abandoned the possibility of “Reply to reply”, an option that was sometimes allowed in the past. Therefore, my reply to the author's reply follows here, perhaps to be followed later with another round of discussion.

In this reply, I'll follow Wisecup's reply closely, like he did with mine. For a proper understanding of my reply-to-reply, it is helpful to have Wisecup's reply at hand.

I'll skip over the first part of the reply, because it is a bit too abstract to react to. It's not contentious anyway. One point though, the only **Dr**.s Vermeer I know of are my father's youngest brother and his daughter.

Assumption of random positions of signal in different sampling realizations. RSI^2 makes use of a (presumed) known spatial relationship of the signal being sampled. In the paper this is the NMO relationship. This known spatial relationship is definitely not random. For nearly all practical purposes it may be “much less an assumption and much more an observation of fact”. Yet, strictly speaking, randomness is an assumption, which deviates from fact, especially for all offsets at deeper levels and for large offsets at shallow levels, where the NMO curve tends to be straight, leading to systematic rather than random sampling.

Requirement of stationarity. Wisecup has a good point that many other seismic processes assume stationarity of the input signal to some degree as well. It's my feeling that RSI^2 needs it more than any other process.

Figure 1 as an explanation of the RSI^2 process. In my discussion I wrote about RSI^2 "…A very clear explanation of what happens…" I did not want to say that Ronen's figure explained the RSI^2 process, but that the idea of combining aliased signals to reconstruct the original signal is not new, and was already illustrated quite nicely by Ronen's figure. The RSI^2 process is of course better illustrated by Wisecup's new figure. To use NMO as the relationship to be used in reconstructing the original signal is probably new (although Neidell does the same with migration).

RSI^2 treats every sample as a separate entity, hence creates high frequencies. This is an assertion I make, and Wisecup disagrees. Why not, he asks, "use the dense cloud of samples on adjacent traces to make a higher quality, multi-channel estimate, free of aliasing". Indeed, that is the intention of RSI^2, but what happens in practice? Individual samples are picked up and put into an output sequence. Samples in the output sequence find themselves surrounded by samples with which they have no direct relationship, their only relationship is the NMO equation. As a consequence, the output sequence will be jittery, and jitter on samples translates itself into spurious frequencies. A smooth function may be fitted to the samples, with the jitter representing unrelated spikes which cover the whole range of frequencies. Therefore, it would have been better to say that random noise with a broad frequency spectrum is generated by the process, rather than high frequencies only. My point is illustrasted with the second graph in Figure 7a of the paper. This graph clearly shows the jitter that is inherent in this process. In this example the jitter is due to out-of-sequence sampling of the noise-free wavelet. If there were noise on the data as well, the jitter would become much larger, and that's why I also suggested that not showing results with noisy data was an omission.

Output samples have to be collected and turned into a single output sample. Wisecup replies this is incorrect: In fact, it would be possible to keep all samples (as individual entities!), and process them through to the final product. Well, in all examples of his paper, he applies the smoothing process, and it would be quite unrealistic to do otherwise.

Is RSI^2 an attractive de-aliasing procedure? In my discussion I point out that the result of 4 ms RSI^2 shown in Figure 10d is inferior to the result of conventional 1 ms processing shown in Figure 10a. According to Wisecup, the important achievement is the improvement of Figure 10d over Figure 10c (4 ms conventional processing). Indeed, this comparison shows that RSI^2 is an interesting de-aliasing procedure with which otherwise lost frequencies can be recuperated under favourable circumstances. However, the phase errors in the result, which are an inevitable product of the process, make the process inferior as compared to alias-free sampling of which the product is shown in Figure 10a. Therefore, I disagree with the statement in the paper "The method has the additional benefit of obviating antialias filtering and sample interpolation." In actual fact, the phase errors in the result demonstrate that the signal cannot be unambiguously recovered with the RSI^2 process (a claim implied by the title of the paper).

RSI² only applies for NMO correction. Wisecup: "Vermeer incorrectly states that RSI² methodology only applies for NMO correction." Careful reading of the last para of my discussion note shows that I mean that the same technique as discussed by Wisecup for NMO is applied by Neidell for migration. It is just the same methodology applied to a different process.

(Neidell's claim "according to the Huygens' approach, achievable resolution can be increased almost without limit if we increase the redundancy of the wavefield sampling", Neidell, 1997, TLE, p. 1414, is based on the interpretation as genuine signal of the spurious high frequencies as also seen in Wisecup's results. It is interesting to note that both scientists have patented their idea.) *Voorschoten, 29 May 1999*