Interpolation beyond Nyquist

Various authors have claimed that their processing technique can extend information beyond Nyquist. Perhaps we should distinguish a few different situations here, and see how sometimes startling results can be explained.

1. Sparse data sets,
2. Wavefield properties allow wavefield reconstruction,
3. Wavenumber spectrum allows reconstruction of most of the data, but not all.

Sparse data sets

In general, synthetic data consist of a limited number of events. Then, with prediction error filtering (Spitz, 1991, Geophysics, p.785), it is possible to reconstruct the data "beyond Nyquist". In such cases we have added information, which can be used to solve the problem. Real data have in general an infinity of different events, then the reconstruction may still look quite good, but it cannot be complete: some of the events may be extrapolated beyond Nyquist, but not all. Of course, doubling the sampling density does not move data beyond Nyquist if the data were not aliased, then it may just improve the visual appearance of the data.

Even with synthetic data, interpolation cannot take place without limits. The normal assumption is local linearity of the events, hence curved events cannot be interpolated with great accuracy across a larger number of traces. Doubling the spatial sampling density often works fine; when trying to step beyond this, the results tend to become visibly worse.

Wavefield properties allow wavefield reconstruction

Sometimes use may be made of properties of the seismic wavefield to de-alias the data. An example is de-aliasing of CMPs for which we may assume that all events have positive time-dip. Then one half of the f-k domain is not used, and may be exploited for recovery of undersampled data in the CMPs of 2D lines (Vermeer, 1990, Seismic Wavefield Sampling, p.87).

Another example is the sampling paradox for 2D data (Vermeer, ib. p.49). Data in CMPs is undersampled whereas it may be well-sampled in the common-shot gathers and in the common-receiver gathers. Via interpolation in the shot and receiver domains, the CMP can be de-aliased. Jakubowicz has applied a combination of the above two procedures for his wavefield reconstruction (SEG Exp. Abstracts, 1994, p. 1557).

These applications exploit the N-dimensional sampling theorem of Petersen and Middleton (1962, Information and Control, p.279) which states that the most efficient sampling lattice is not in general rectangular. Sampling 3D common-receiver gathers using shots in a hexagonal arrangement rather than a square arrangement is another application of that N-D theorem.

Wavenumber spectrum allows reconstruction of most of the data, but not all

Often, the wavenumber spectrum of the seismic data is heavily skewed toward the small wavenumbers, with only a relatively small amount of energy present along the high wavenumbers. In this situation, the bulk of the data may be properly sampled, with little aliasing being visible. Interpolation to the double sampling density will be quite successful on the bulk of the data, and the little aliasing there is does not lead to strong artifacts. This situation almost always plays a role, also in combination with the situations described above. A benefit from this interpolation may also lie in the larger number of samples for each wavelength. This may make interpretation much easier, even though the original sampling was OK.

Yet, the resolution of the data is determined by the highest unaliased wavenumbers. These describe the behavior of diffractions from reflector terminations against faults, and the fault planes themselves. Interpolation will not help this information, if it is aliased. This will be most clearly visible on dip or dip azimuth maps which are derived from horizon slices by taking derivatives.

The same interpolation techniques applied to steep geology are bound to break down.

Summary

Claims that certain processes can reconstruct the wavefield “beyond Nyquist” should be handled with caution. There is always an explanation using existing sampling theory. Often the techniques are quite useful, as they improve the appearance of the data and even improve the results of later processing steps, but they should not be used as an excuse to relax sampling requirements.